IL NUOVO CIMENTO **43 C** (2020) 45
DOI 10.1393/ncc/i2020-20045-0

COLLOQUIA: IFAE 2019

General correlations to $b \to s\mu^+\mu^-$ anomalies from a rank condition

V. GHERARDI(1)(2)(*)

(1) *SISSA International School for Advanced Studies - Via Bonomea 265, 34136, Trieste, Italy*
(2) *INFN, Sezione di Trieste - Via Bonomea 265, 34136, Trieste, Italy*

received 8 June 2020

Summary. — It is assumed that the New Physics addressing neutral-current B-meson anomalies couples to a single direction in quark flavor space, *i.e.*, that the Wilson coefficient matrix of the relevant semi-leptonic operators be of rank one. By correlating the observed anomalies to other flavor and high-p_T observables, we constrain the possible flavor directions involved in our assumption.

1. – Introduction

The deviations from Standard Model (SM) predictions observed in $b \to s\mu\mu$ transitions represent, to date, one of the few hints of New Physics (NP) living at, or near, the TeV scale. These so-called *anomalies* have been reported in several independent observable measurements, including Lepton Flavor Universality (LFU) ratios $R(K)$ [2,3] and $R(K^*)$ [4,5], differential branching fractions in $b \to s\mu\mu$ transitions [6], angular distributions in $B \to K^*\mu^+\mu^-$ [7-9] and the leptonic decay branching fraction $\mathrm{Br}(B_s^0 \to \mu^+\mu^-)$ [10,11]. For a nice experimental overview, see ref. [12].

What makes these results particularly intriguing is that all deviations can be explained by a single NP contribution to one of the following semimuonic operators(1):

$$\mathcal{O}_L = (\bar{s}\gamma_\rho P_L b)(\bar{\mu}\gamma^\rho P_L \mu), \qquad \mathcal{O}_9 = (\bar{s}\gamma_\rho P_L b)(\bar{\mu}\gamma^\rho \mu), \tag{1}$$

which can be thought to be part of an effective Lagrangian involving all three quark families,

$$\mathcal{L}_{\mathrm{NP}}^{\mathrm{EFT}} = C_L^{ij}(\overline{d_i}\gamma_\rho P_L d_j)(\bar{\mu}\gamma^\rho P_L \mu) + C_R^{ij}(\overline{d_i}\gamma_\rho P_L d_j)(\bar{\mu}\gamma^\rho P_R \mu) \tag{2}$$

(*) Based on work in collaboration with D. Marzocca, M. Nardecchia and A. Romanino [1].
(1) For explanations involving NP in semielectronic operators, see [13].

(we focus here on processes involving muons on the leptonic side). Assuming the $b \to s\mu\mu$ anomalies to be a genuine NP effect, from the point of view of eq. (2), it is natural to ask whether the same NP may affect other semileptonic channels, such as $d_i \to d_j\mu\mu$ (or more generally $q_i \to q_j\ell_\mu\ell_\mu$, with q_i and ℓ_μ being an up or down quark and a muon or a muonic neutrino, respectively). Clearly, in order to establish such *correlations*, one requires additional information concerning the flavor structure of the matrices $C^{ij}_{L,R}$.

In this paper, I describe a framework, Rank-One Flavor Violation [1], in which the correlations arise from a rank condition satisfied by the Wilson Coefficients (WC) matrices. In sect. **2** I detail and motivate such a flavor assumption, and present its most general implications; in sect. **3** I promote eq. (2) to a full $SU(2)_L$ invariant Lagrangian and examine its consequences; in sect. **4** I make contact with approaches based on UV flavor symmetries; conclusions are drawn in sect. **5**.

2. – Setup and general correlations

It is assumed that the WCs $C^{ij}_{L,R}$ in eq. (2) are of rank-one and proportional, that is to say

$$(3) \qquad C^{ij}_{L,R} = C_{L,R}\hat{n}_i\hat{n}^*_j, \qquad \hat{n} = \begin{pmatrix} \sin\theta\cos\phi e^{i\alpha_{bd}} \\ \sin\theta\sin\phi e^{i\alpha_{bs}} \\ \cos\theta \end{pmatrix}.$$

Here $\hat{n}$ is a unit vector in quark flavor space, and $C_{L,R}$ are real numbers.

From the physical point of view, this amounts to say that the NP sector responsible of the $b \to s\mu\mu$ signal couples to a single direction in quark flavor space, *i.e.*, $\mathcal{L}_{\rm NP} = \mathcal{L}_{\rm NP}[\hat{n}^*_i q_i]$. In turn, this is realized in a manifold of UV models, to cite some examples: models with a single leptoquark(2) (see ref. [14] for a recent comprehensive review); models in which the quark doublet mixes with a single generation of vector-like fermions; 1-loop models with linear flavor violation [15]. In all these scenarios, it makes indeed sense to ask: *what is the direction* $\hat{n}$ *of NP*?

Under the assumptions in eq. (3), the effective Lagrangian (2) depends upon a scale C_L, two angles θ and ϕ and two phases α_{bd} and α_{bs} in the definition of $\hat{n}$, and the relative weight C_R/C_L. As an additional working hypothesis, I consider the two cases $C_R = 0$ and $C_R = C_L$, which are again well-motivated benchmarks from an high-energy point of view. A fit to $b \to s\mu\mu$ observables allows then to determine the phase $\alpha_{bs} \approx 0$ and the scale C_L, and thus the whole effective Lagrangian (2) as a function of the quark direction $\hat{n}$ (see ref. [1] for details). Comparison with experimental values/bounds relative to other $d_i \to d_j\mu\mu$ channels tells whether a given NP direction $\hat{n}$ is excluded or not.

The results for $C_R = 0$ and $\alpha_{bd} = 0, \pi/2$ are shown in fig. 1, where I display the bounds coming from the $d_i \to d_j\mu\mu$ observables reported in the legends. As one can see, directions with a sizable component along the first family are largely disfavoured, whereas a generous region with $\hat{n} \approx (\mathcal{O}(V_{td}), \mathcal{O}(V_{ts}), 1)^t$ is allowed (red-shaded region in

(2) Strictly speaking, the correlations discussed in the present work apply to all single leptoquark models in which the coupling to electrons is suppressed with respect to the one to muons.

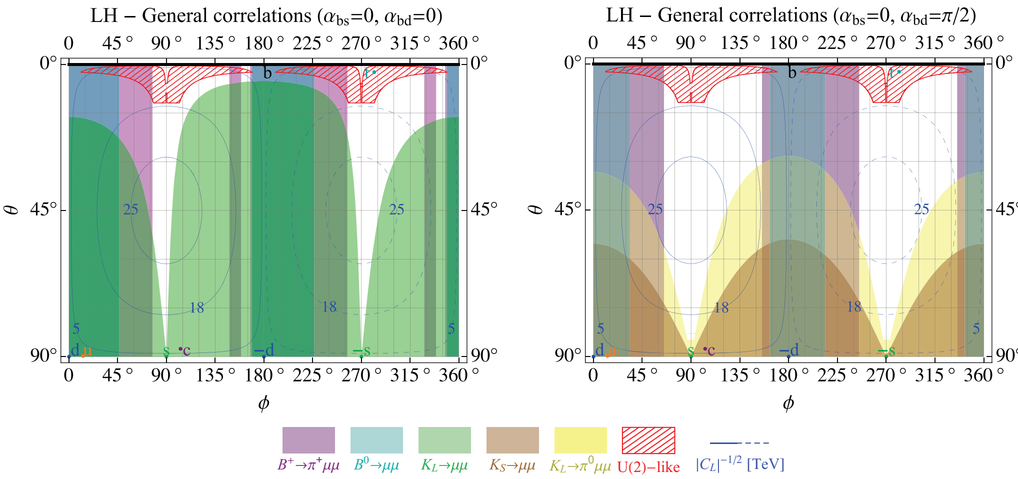

Fig. 1. – Limits in the plane (ϕ, θ) for two choices of the phases α_{bs} and α_{bd} from observables with direct correlation with $R_{K^{(*)}}$. The blue contours correspond to the value of $|C_L|^{-1/2}$ in TeV, where solid (dashed) lines are for positive (negative) C_L. The meshed red region correspond to the one suggested by $SU(2)_q$-like flavor symmetry, cf. sect. **4**.

the figure). This region is also theoretically favoured from the point of view of UV flavor symmetries (see sect. **4**).

3. – SU$(2)_L$ invariance and simplified mediators

Assuming that the relevant NP degrees of freedom lie above the electroweak scale, the natural framework for model-independent studies of the anomalies is actually that of the Standard Model Effective Field Theory (SMEFT). The SMEFT operators that can contribute to the above low-energy ones at the tree level are collected in the following Lagrangian:

$$\begin{aligned}\mathcal{L}_{\rm NP}^{\rm SMEFT} &= C_S^{ij}(\overline{q_{iL}}\gamma_\mu q_{jL})(\overline{\ell_{2L}}\gamma^\mu \ell_{2L}) + C_T^{ij}(\overline{q_{iL}}\gamma_\mu \sigma^a q_{jL})(\overline{\ell_{2L}}\gamma^\mu \sigma^a \ell_{2L}) \\ &\quad + C_R^{ij}(\overline{q_{iL}}\gamma_\mu q_{jL})(\overline{\mu_R}\gamma^\mu \mu_R),\end{aligned} \tag{4}$$

giving $C_L^{ij} = C_S^{ij} + C_T^{ij}$ in eq. (2). In the previous equation, $\ell_L^i = (\nu_L^i, e_L^i)^t$ and $q_L^i = (V_{ji}^* u_L^j, d_L^i)^t$ are the lepton and quark doublets, in the charged-lepton and down quarks mass basis, respectively, and V is the CKM matrix. Under the ROFV assumption, we have (cf. eq. (3)) $C_{S,T,R}^{ij} = C_{S,T,R}\hat{n}_i\hat{n}_j^*$.

Contrary to what one may naively expect, simply promoting the low-energy effective Lagrangian (2) to an $SU(2)_L$ invariant Lagrangian does not give rise to new correlations. This is most immediately realized by looking at table I, where it is shown the dependence of the various semileptonic processes upon $C_{S,T,R}$. The phenomenologically relevant FCNC processes $u_i \to u_j\mu\mu$ and $d_i \to d_j\nu\nu$ depend upon the combination $C_S - C_T$, which is not fixed by $b \to s\mu\mu$, whereas the FCNC $u_i \to u_j\nu\nu$ and FCCC $d_i \to u_j\mu\nu$ processes do not give rise to appreciable constraints.

TABLE I. – *Dependencies of various semileptonic processes on the three coefficients $C_{S,T,R}$ (cf. eq. (4)). Here and in the text, a given quark level process represents all processes obtained through a crossing symmetry from the shown one.*

Channel	Coefficient dependencies
$d_i \to d_j \mu^+\mu^-$	$C_S + C_T,\ C_R$
$u_i \to u_j \overline{\nu_\mu}\nu_\mu$	$C_S + C_T$
$u_i \to u_j \mu^+\mu^-$	$C_S - C_T,\ C_R$
$d_i \to d_j \overline{\nu_\mu}\nu_\mu$	$C_S - C_T$
$u_i \to d_j \mu^+\nu_\mu$	C_T

In order to fully exploit $SU(2)_L$ invariance, thus, it is necessary to fix the ratios $C_S\colon C_T\colon C_R$. A sensible way to do this is by assuming that the effective operators in eq. (4) are generated by the exchange of a single mediator with specific quantum numbers. In fig. 2 I show the results for the scalar leptoquark $S_3 \sim (\overline{3}, 3, 1/3)$, whose tree-level

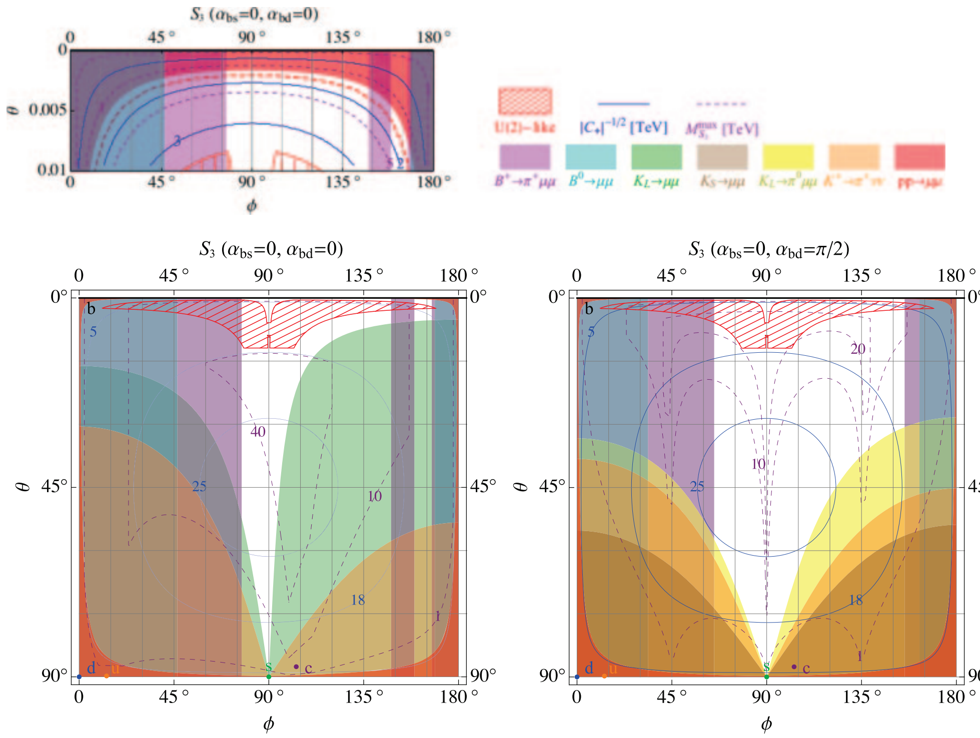

Fig. 2. – Limits in the (ϕ, θ) plane for the scalar leptoquark S_3 and two choices of the phases α_{bs} and α_{bd}. In addition to the limits in fig. 1, the orange bound is from $K \to \pi\nu\nu$ while the red one is from the high-p_T tail of $pp \to \mu^+\mu^-$ at the LHC [16]. The top-left panel is a zoom of the region $\theta \ll 10$ of the bottom-left one, which shows in more detail the region excluded by LHC dimuon searches. (The dashed purple contour lines are upper limits, in TeV, on the leptoquark mass from $\Delta F = 2$ processes; see ref. [1] for details.)

exchange gives rise to $C_S : C_T : C_R = 3 : 1 : 0$. In addition to the general bounds of fig. 1, there are two bounds coming from the $d_i \to d_j \nu\nu$ and flavor observables listed in the legends; for the sake of comparison, I also display a collider bound coming from the high-p_T tails of $pp \to \mu^+\mu^- X_{\text{had}}$ at LHC [16], which is *de facto* weaker than flavor bounds in almost all parameter space.

4. – Flavor symmetries and $U(2)^5$

In the previous sections, I have been agnostic about the direction of the unit vector $\hat{n}$. Here I would like to illustrate some possible theoretical expectations, based on a high-energy flavor symmetry example.

It is assumed that a subgroup $\mathcal{G}$ of the SM flavor symmetry group, *i.e.*, $U(3)^5 \equiv \prod_{f=q,u,d,\ell,e} U(3)_f$, be an actual UV symmetry, which must spontaneously broken in the IR by the expectation values of a suitable set of spurions $\mathcal{S}_\mathcal{G}$. Indeed, the first requirement on $\mathcal{S}_\mathcal{G}$ is that it must be possible to reproduce the SM Yukawa $y_{U,D,E}$ representations under $\mathcal{G}$. In addition, in order to be compatible with our rank-one NP scenario, it has to be possible to reproduce the WCs $C_{S,T,R}$ representations.

The simplest example is that of Minimal Flavor Violation (MFV) [17], which is defined by $\mathcal{G} = U(3)^5$ and $\mathcal{S}_\mathcal{G} = \{y_U \sim 3_q \otimes \overline{3}_u,\, y_D \sim 3_q \otimes \overline{3}_d,\, y_E \sim 3_\ell \otimes \overline{3}_e\}$. Under the MFV group, one has $C_{S,T,R} \sim (1 \oplus 8)_q \otimes \rho_L$, where ρ_L is some $U(3)_\ell \times U(3)_e$ representation. With the spurions at disposal, the WCs $C_{S,T,R}$ can be constructed as functions of $y_U y_U^\dagger$ and $y_D y_D^\dagger$ and respective traces; barring fine-tunings, $C_{S,T,R}$ are generically rank three matrices, contrary to the rank-one assumption.

Another well-motivated example consists of $\mathcal{G} = U(2)^5$, where the $U(2)$ factors act on light generation fermions, extending the quark $U(2)^3$ [18]. Here $\mathcal{S}_\mathcal{G}$ must include a doublet $V_q \sim 2_q$ which can be identified with $V_q = (V_{td}^*, V_{ts}^*)$ in the down-aligned basis. Under the minimal assumption that no other spurion charged under both the quark $U(2)^3$ and the lepton $U(2)^2$ exists, one can show that $\hat{n} \propto (c e^{i\gamma} V_q, 1)^t$, where c is an $\mathcal{O}(1)$ real number. It turns out that, for any $-60^\circ \lesssim \gamma \lesssim 60^\circ$ and $c \ll 20$, the corresponding $\hat{n}$ is allowed by the flavor bounds considered in the previous Sections, showing a good compatibility between the ROFV assumption and $U(2)^5$ flavor symmetry. Concerning the observables under study, it is also worth to point out two interesting relations:

$$R(K) \approx R(\pi), \qquad \frac{\text{Br}(B_s^0 \to \mu^+\mu^-)}{\text{Br}(B_s^0 \to \mu^+\mu^-)^{\text{SM}}} \approx \frac{\text{Br}(B_d^0 \to \mu^+\mu^-)}{\text{Br}(B_d^0 \to \mu^+\mu^-)^{\text{SM}}}, \tag{5}$$

which are valid, up to few percents corrections, under the minimally broken $U(2)^5$ symmetry assumption only (*i.e.*, irrespectively of the rank one structure)([3]). I address the interested reader to ref. [1] for further details concerning the link with flavor symmetries.

5. – Conclusions

If the $b \to s\mu\mu$ anomalies will be experimentally confirmed, studying their correlations with other flavor observables will represent a powerful tool to understand the structure of New Physics. In this contribution, I have described a framework, Rank-One

([3]) I would like to express my gratitude to M. Bordone for bringing this point to my attention.

Flavor Violation, motivated by explicit UV models, where correlations stem from a rank condition.

I have presented some key observables which sizably narrow down the ROFV parameter space. This provides a fully data driven input for models realizing a rank-one flavor structure, which turns out to be well compatible with theoretical expectations motivated by $U(2)^5$ flavor symmetry.

REFERENCES

[1] Gherardi V., Marzocca D., Nardecchia M. and Romanino A., *JHEP*, **10** (2019) 112, arXiv:1903.10954 [hep-ph].

[2] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **113** (2014) 151601, arXiv:1406.6482 [hep-ex].

[3] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **122** (2019) 191801, arXiv:1903.09252 [hep-ex].

[4] LHCb Collaboration (Aaij R. *et al.*), *JHEP*, **08** (2017) 055, arXiv:1705.05802 [hep-ex].

[5] Belle Collaboration, *Search for $B \to l\nu\gamma$ and $B \to \mu\nu$ and test of lepton universality with $R(K^*)$ at Belle*, in *Proceedings of the 54th Rencontres de Moriond on Electroweak Interactions and Unified Theories (Moriond EW 2019), La Thuile, Italy, 16–23 March 2019*.

[6] LHCb Collaboration (Aaij R. *et al.*), *JHEP*, **06** (2014) 133, arXiv:1403.8044 [hep-ex].

[7] LHCb Collaboration (Aaij R. *et al.*), *JHEP*, **09** (2015) 179, arXiv:1506.08777 [hep-ex].

[8] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **111** (2013) 191801, arXiv:1308.1707 [hep-ex].

[9] LHCb Collaboration (Aaij R. *et al.*), *JHEP*, **02** (2016) 104, arXiv:1512.04442 [hep-ex].

[10] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **118** (2017) 191801, arXiv:1703.05747 [hep-ex].

[11] ATLAS Collaboration (Aaboud M. *et al.*), *JHEP*, **04** (2019) 098, arXiv:1812.03017 [hep-ex].

[12] Borsato M., this conference.

[13] Iyer A. M., this conference.

[14] Angelescu A., Beirevi D., Faroughy D. A. and Sumensari O., *JHEP*, **10** (2018) 183, arXiv:1808.08179 [hep-ph].

[15] Gripaios B., Nardecchia M. and Renner S. A., *JHEP*, **06** (2016) 083, arXiv:1509.05020 [hep-ph].

[16] Greljo A. and Marzocca D., *Eur. Phys. J. C*, **77** (2017) 548, arXiv:1704.09015 [hep-ph].

[17] D'Ambrosio G., Giudice G. F., Isidori G. and Strumia A., *Nucl. Phys. B*, **645** (2002) 155, arXiv:hep-ph/0207036.

[18] Barbieri R., Buttazzo D., Sala F. and Straub D. M., *JHEP*, **07** (2012) 181, arXiv:1203.4218 [hep-ph].