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Flavour neutrino oscillations in matter moving with acceleration

A. STUDENIKIN[1][2], I. TOKAREV[1]

[1] Department of Theoretical Physics, Faculty of Physics, Moscow State University - Moscow
119991, Russia
[2] Joint Institute for Nuclear Research - Dubna 141980, Moscow Region, Russia

Summary. — Neutrino flavour oscillations in a nonuniformly moving matter are considered. The neutrino oscillation resonance condition in presence of matter, in the most general case when matter is moving with acceleration, is derived for the first time. We predict that the effect of matter acceleration can have significant influence on neutrino oscillations pattern in different astrophysical environments.

In this short note we generalize our previous studies [1] on neutrino flavour oscillations in an uniformly moving matter to the case when matter moves with acceleration.

Consider two flavour (electron and muon) neutrino oscillations in nonuniformly moving matter. The effective Lagrangian of neutrino interactions with background matter can be expressed in the form

$$(1) \qquad \mathcal{L}_{eff} = -\bar{\nu}\left(\gamma_\mu f^\mu \frac{1+\gamma^5}{2}\right)\nu, \quad f^\mu = \frac{G_F n \gamma}{\sqrt{2}}(1, \boldsymbol{v}),$$

where G_F is the Fermi constant, $\boldsymbol{v}$ is the speed of matter, $\gamma = (1-\boldsymbol{v}^2)^{-\frac{1}{2}}$ and $n = n_0$ and $n = -n_0$ for the electron and muon neutrino correspondingly. Here n_0 is the electron number density in the reference frame for which the total speed of matter is zero. Note that we consider the matter composed of neutrons, protons and electrons. Obviously, neutrons and protons do not influence the oscillations.

For the effective Hamiltonian of neutrino flavour states evolution we obtain

$$(2) \qquad H = \sigma_1 \frac{\delta m^2 \sin 2\theta}{4E} - \sigma_3\left(\frac{\delta m^2 \cos 2\theta}{4E} - \frac{G_F n_0}{\sqrt{2}}\frac{1-v(x)\cos\phi}{\sqrt{1-v^2(x)}}\right),$$

where $\delta m^2 = m_2^2 - m_1^2$, θ is the vacuum mixing angle, E is the neutrino energy and ϕ is the angle between the speed of matter and the direction of neutrino propagation (it is supposed that the neutrino is propagating along x direction). In the adiabatic approximation the oscillation probability has the usual form $P_{\nu_e \to \nu_\mu}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}}$,
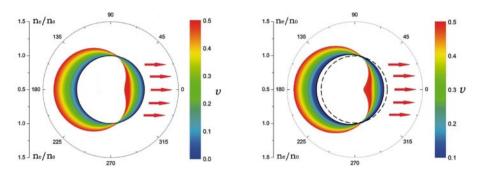
Fig. 1. – The figure illustrates distribution of the effective electron density in moving matter (normalized on the static value) as a function of matter speed v and the angle of propagation ϕ. The red arrows define the direction of the neutrino flux. The left and right plots corresponds to $v_0 = 0$ and $v_0 = 0,1$ accordingly.

where the effective mixing angle θ_{eff} and oscillation length L_{eff} are determined by the elements H_{ij} of the evolution Hamiltonian (2).

The straightforward calculations yields the neutrino flavour oscillations resonance condition [2,3]

$$(3) \qquad \frac{\delta m^2}{2E} \cos 2\theta = \sqrt{2} G_F n_e(x), \quad \text{where} \quad n_e(x) = \frac{n_0}{x} \int_0^x \frac{1 - v(x') \cos \phi}{\sqrt{1 - v^2(x')}} dx'.$$

In the considered case the neutrino oscillations probability gets its maximum value in a set of points x_k that are the solutions of Eq. (3). Note that in general case Eq. (3) is not linear in respect to x. Obviously, that in case of monotonic x dependence of the density n_e there can be only one resonance point.

Consider matter motion with a constant acceleration a. Then the matter velocity is given by $v(x) = \frac{v_0 \gamma_0 + ax}{\sqrt{1 + (v_0 \gamma_0 + ax)^2}}$ where v_0 is the initial matter speed. In this case one can obtain the effective electron matter density in the form

$$(4) \qquad n_e = \frac{n_0}{2} \left(\frac{\gamma V - \gamma_0 V_0 + \ln \frac{\gamma + V}{\gamma_0 + V_0}}{V - V_0} - (V + V_0) \cos \phi \right), \quad V \equiv \gamma v.$$

In Fig. 1 the function $n_e = n_e(v(x), \phi)$ is plotted. It follows that n_e significantly increases in case of matter motion against neutrino propagation ($\phi = \pi$) and almost vanishes in the opposite case ($\phi = 0$). In the nonrelativistic and ultrarelativistic limits of matter motion Eq. (4) can be simplified as follows

$$(5) \qquad \frac{n_e}{n_0} = \begin{cases} (4ax)^{-1}, & (ax \gg 1, \phi = 0), \\ ax \sin^2 \frac{\phi}{2}, & (ax \gg 1, \phi \neq 0), \\ 1 - \frac{ax}{2} \cos \phi, & (ax \ll 1). \end{cases}$$

In the corresponding evaluation it is supposed that $v_0 = 0$. When nonrelativistic matter is moving with a constant acceleration the corresponding shift of the electron number density is given by $\delta n(x) = \frac{1}{2} n_0 \delta v(x) \cos \phi$, where $\delta n(x) = n_e(x) - n_0$ and $\delta v(x) = v(x) - v_0$. For instance, during a supernova core-collapse $\delta v(x)$ is up to 0,2 that yields an order of 10% shift to the effective resonance number density n_e. It would be interesting to consider possibilities of observing this effect.

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