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Colloquia: Pontecorvo100

Theory of neutrino masses and mixing

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Summary. — In spite of the enormous experimental progress in the determination of the neutrino parameters, a theory of neutrino mass and mixing is still on the cross-roads. Guidelines could be (i) the connection between zero neutrino charges (and therefore a possibility to be Majorana particle), smallness of the neutrino mass and large lepton mixing, (ii) joint description of leptons and quarks, (iii) existence of the right handed (RH) neutrinos without special quantum numbers. Properties of the RH neutrinos and the UV completion of the seesaw may turn out to be the key to understand the neutrino mass and mixing. In view of the LHC results minimalistic scenarios like νMSM look rather plausible. Still the GUT's with additional hidden sector, QLC, high scale flavor symmetries are appealing. Concerning mixing, the main issue is "symmetry or no symmetry" behind the observed pattern. The symmetry group condition is a useful tool to study the consequences of symmetries and to perform "symmetry building". Sterile neutrinos are a challenge but also opportunity for the present theoretical constructions.

1. – Introduction

The title of this talk is a joke in the spirit of Bruno Pontecorvo. In reality we have plenty of observations, mechanisms, schemes, models, approaches, conjectures and even scans of various possibilities: symmetries, parameters, field contents (see reviews [1]).

It is widely accepted (which does not mean much) that new physics beyond the standard model and beyond just adding of the right handed neutrinos is involved. However, the proposed mass scales of this new physics range from eV to the Planck mass, that is, 28 orders of magnitude. For explanation of mixing a spectrum of ideas spans from symmetry to anarchy [2]. This means that we are far from real understanding of the underlying physics. As a consequence, we can not predict unambiguously mass hierarchy, the value of CP phase, the absolute scale of neutrino mass, *etc.*.

Frameworks under discussion cover minimalistic phenomenological scenario of νMSM on one extreme, and sophisticated structures at different energy scales on another. Studies spread from simple-minded manipulations with mass and mixing matrices to consideration of geometric, strings as well as complicated dynamics origins of the observed pattern. Certain models and approaches can indeed correspond to reality, and still some key elements may be missed.

Recent trends are determined by the fact that no new physics has been discovered at LHC and other experiments. Higgs boson properties are in agreement with those of the SM. This hints that we should be stingy in our speculations, and take more seriously scenarios with nothing or almost nothing up to the Planck scale.

In this connection some ideas, approaches and results will be reviewed which may have a chance to reflect reality. I will consider problems of construction of the theory of neutrino mass and mixing.

2. – Facts and feelings

The data from now numerous solar, atmospheric, reactor, accelerator neutrino experiments can be nicely described in the 3ν mixing framework. All the mixing angles, θ_{ij}, as well as mass squared differences Δm_{21}^2 and $|\Delta m_{31}^2|$ are determined with rather good accuracy [3]. As far as theory is concerned, even after precise measurements of the 1-3 mixing we are at the cross-roads. The same value of 1-3 mixing has different relations to other parameters with completely different implications. The most appealing possibilities are

1. "Naturalness" - absence of fine tuning in the mass matrix gives [4]:

$$\sin^2\theta_{13} = A\frac{\Delta m_{21}^2}{\Delta m_{31}^2}, \qquad A = \mathcal{O}(1). \tag{1}$$

2. Connection to Cabibbo mixing [5]:

$$\sin^2\theta_{13} \approx \frac{1}{2}\sin^2\theta_C, \tag{2}$$

which can be realized in the context of Quark-Lepton Complementarity [6] with implications of GUT or/and family symmetry [7].

3. Connection to deviation of 2-3 mixing from maximal:

$$\sin^2\theta_{13} \approx \frac{1}{2}\cos^2 2\theta_{23} \quad \text{or} \quad \theta_{13} \approx \sqrt{2}(\pi/4 - \theta_{23}), \tag{3}$$

which was predicted in model with T' symmetry [8] but may also follow from the universal $\nu_\mu - \nu_\tau$ symmetry violation [9].

4. Inter-generation connection:

$$\sin^2\theta_{13} \approx \frac{1}{4}\sin^2\theta_{12}\sin^2\theta_{23}, \tag{4}$$

which is analogous to the quark relation $V_{ub} = 0.5V_{us}V_{cb}$ (the $q - l$ similarity). This may follow from a kind of Fritzsch texture for mass matrices (with texture zeros, U(1) symmetry, etc.). For more cases see [9].

Determination of other unknowns (mass ordering, CP violation phase, absolute scale of mass) may or may not help depending on the outcome. E.g., in the case of normal mass ordering possible hierarchy is weak or absent: $m_2/m_3 \geq \sqrt{\Delta m_{21}^2/\Delta m_{31}^2}$. Still this resembles the situation in the quark sector. Inverted mass hierarchy implies strong degeneracy: $\Delta m_{21}/m_1 \leq 1.6 \cdot 10^{-2}$, and therefore certain symmetry. Even more symmetry will be realized if the spectrum is degenerate. There are first glimpses of the CP phase: global fits (essentially the atmospheric neutrino data) indicate $\delta_{CP} \sim \pi$ [3].

In view of the present trends it makes sense *to refine and sharpen known arguments,* as actually Pontecorvo did continuously. There are three well established facts about neutrinos:

$$(i) \;\; Q_\gamma = Q_c = 0, \qquad (ii) \;\; m_\nu \ll m_l, m_q, \qquad (iii) \;\; \theta_{12} \sim \theta_{23} \sim 1 \tag{5}$$

and zero values of conserved charges (i) mean that neutrinos can be Majorana particles. Naturally, one expects that these facts are connected.

SM and Weinberg operator. After decoupling of possible heavy degrees of freedom of new physics one obtains in the lowest order the D=5 operator [10]

$$\frac{1}{\Lambda} LLHH, \tag{6}$$

where Λ is the scale of new physics. The operator generates Majorana neutrino mass and to a large extent, realizes the connection (5). Is this the end of "theory of neutrino mass and mixing"? Can we say more and advance further? As *guidelines and prejudices* I would take

- Minimality and connection (5).
- Quark-lepton analogy (correspondence, symmetry, unification). Theory of masses and mixing should include both quarks and leptons, although some new elements may be present in lepton sector.
- Existence of the right-handed neutrinos *without* any special symmetry (new quantum numbers).

3. – RH neutrinos, seesaw and its UV-completion.

Once the RH neutrino components without special quantum numbers are introduced, unavoidably neutrinos should have the Dirac mass terms as all other leptons and quarks have. Then smallness of the mass can be due to very small Yukawa couplings (still to be explained) or some new physics. The natural way to suppress the neutrino Dirac mass is to introduce large Majorana masses of ν_R, realizing the seesaw [11]: $m_\nu = -m_D M_R^{-1} m_D^T$. In minimal version the seesaw simultaneously suppresses the Dirac mass and generates small Majorana mass without introduction of new symmetry. However, seesaw may provide mainly suppression of the Dirac masses, if e.g. the RH neutrino masses are at the Planck scale. Then dominant contribution to m_ν comes from another mechanism.

In the minimal version of seesaw the scale of ν_R masses is

$$M_R \sim m_D^2/m_\nu \sim 10^{14} \text{ GeV}. \tag{7}$$

That is, ν_R introduces new mass scale which is much smaller than M_{PL}. Smallness of neutrino mass is another indication of existence of new physics scale apart from unification of gauge couplings.

The existence of heavy ν_R affects the Higgs sector in two ways:

1. It gives (the loop with ν_l and ν_R) the correction to the Higgs mass which is quadratically divergent [12]:

$$\delta m_H^2 \approx \frac{y^2}{(2\pi)^2} M_R^2 \log(q/M_R) \sim \frac{M_R^3 m_\nu}{(2\pi v)^2} \log(q/M_R), \tag{8}$$

where v is the electroweak VEV and y is the Yukawa coupling. Typically one gets $\delta m_H^2 \sim (10^{13}\ \text{GeV})^2$.

2. It modifies (loop with $\nu_l, \nu_R, \nu_l, \nu_R$) the RG running of the quartic Higgs coupling λ, and consequently, modifies the Higgs potential. The high scale minimum becomes deeper which in turn, affects stability (lifetime) of the EW vacuum [13]. For Higgs masses below 128 GeV the effect is small.

Possible solutions of the problem 1) are

- Cancellation of contribution (8) due to existence of new particles (which is realized, e.g., in SUSY), or fine tuning, in which case the dependence of low scale observables, in particular Higgs mass, on high scale physics parameters (mass of RH neutrino) becomes enormous.

- Reduction of the seesaw scale $M_R \sim M_{EW}$. This requires small Yukawa couplings or cancellation of contributions from different RH neutrinos.

- Increase of the seesaw scale up to the Planck scale $M_R \sim M_{Pl}$ by the prize of introduction of e.g. many RH neutrinos. In this case one can simply "blame" some new Planck scale physics which is responsible for tuning of parameters.

- Modification of the seesaw mechanism: introduction of more degrees of freedom, in particular, more than 3 RH neutrinos. In the double seesaw [14] three additional singlets S with Majorana masses μ couple to (mix with) the RH neutrinos via the Dirac mass term M_D. Then

$$m_\nu = m_D M_D^{-1} \mu M_D^{T-1} m_D^T. \tag{9}$$

There are three possibilities depending on the size of the lepton violating mass μ: (i) $\mu = 0$ which gives one massless neutrino per generation. This is an example of multi-singlet (or "chiral mismatch") mechanism of suppression of the Dirac mass. Physical consequences include violation of universality and unitarity for lights states characterized by the ratio m_D/M_D. (ii) $\mu \ll M_D$ corresponds to the inverse seesaw. It allows to lower the scales of neutrino mass generation and still have large enough probability of production of new heavy states at LHC. The spectrum of the heavy states is composed of pseudo-Dirac heavy leptons with small mass splitting. For $M_D \sim$ TeV, one has $\mu \sim$ kev which can be generated radiatively [15]. (iii) $\mu \gg M_D$ - cascade seesaw. This leads to masses of the RH neutrinos $M_R \sim M_D^2/\mu$.

The ultraviolet completion of the high scale seesaw can be one of the driving forces of new developments. Interestingly, other mechanisms are also related to properties of the RH neutrinos. Thus, in the case of extra spatial dimensions [16], [17] the overlap mechanism works with different localizations of the left and right handed neutrino components.

4. – Large mass scales, small mass scales and LHC

Low scale mechanisms of neutrino mass generation can, in principle, be tested at LHC and other laboratory experiments. Some possibilities include.

1. Radiative mechanisms. The main features of e.g. the one loop mechanism from [18] are (i) absence of usual RH neutrinos; (ii) new Higgs doublet (η^+, η^0), (iii) fermionic singlets N_k. They are odd under discrete symmetry Z_2, whereas the SM particles are Z_2 even; η^0 has zero VEV. If Z_2 is exact, η^0 or the lightest N_k are stable and can be the Dark Matter particles. So, here a popular neutrino mass - DM connection is realized.

Two loops Zee-Babu mechanism and its modifications are still "on the market" [19]. Models have no RH neutrinos, new scalar singlets h^- and k^{++} are introduced. The models are testable: new charged bosons can be produced at LHC, decays $\mu \to \gamma e$, and $\tau \to 3\mu$ are predicted with rates within reach of forthcoming experiments.

2. Smallness of neutrino mass can be due to new Higgses, e.g., Higgs triplet in the seesaw type II or new Higgs doublets with small VEV's.

3. Low scale L-R symmetry models with $M(W_R) \sim$ few TeV and $M(N_R) \sim$ 0.5 - few TeV [20] have several different contributions to the light neutrino masses, including the see-saw type I with small Yukawa couplings, Higgs triplet mechanism. Signatures of the models at LHC are the same-sign bi-leptons, $lljj$, and no missing energy [21]. For $M(W_R) > M(N_R)$ resonance production of W_R occurs. Peaks at the invariant mass $W(jjl) = m_N$, and $W(jjll) = m_W$ should be observed. In the t-channel the corresponding diagram coincides with the diagram for $0\nu\beta\beta$ decay. Consequently, complementary bounds from LHC and $0\nu\beta\beta$ can be obtained [22].

The νMSM scenario [23] deserves now special attention. Its signature is that nothing should be seen at LHC. Everything is at or below the EW scale including masses of the RH neutrinos. Consequently, small Yukawa couplings should be introduced. Spectrum of the model consists of a) two strongly degenerate states with masses $\sim (0.1 - 5)$ GeV, and splitting $(10^{-3} - 1)$ eV. They generate light masses of neutrinos via seesaw, and the lepton asymmetry in the Universe via $\nu-$oscillations. They can be produced in $B-$meson decays with BR $\sim 10^{-10}$. b) One RH neutrino with mass (3 - 10) kev and very small mixing with active neutrinos plays the role of warm dark matter. It can be searched for through its radiative decay as an X-ray line [23]. The Higgs inflation scenario can be realized [24] which is in a good agreement with the Planck data.

High scale seesaw mechanisms can not be probed at LHC either. Here there are two interesting realizations: 1). GUT seesaw with $M_R \sim M_{GUT} \sim 10^{16}$ GeV which is possible for the heaviest RH neutrino. Leptogenesis is due to the CP-violating out of equilibrium decay of RH neutrinos. 2). Double (cascade) seesaw with $\mu \sim M_{Pl}$ and $M_D \sim M_{GUT}$ explains the intermediate scale (7) for the RH neutrinos $M_R \sim M_{GUT}^2/M_{Pl} \sim 10^{14}$ GeV.

An appealing scenario is the SO(10) GUT with 16-plet fermions, hidden sector at GUT - Planck scales composed of fermion and scalar singlets of SO(10). The presence of the fermion singlets with addition of the Higgs 16-plet can realize high mass scale double seesaw, enhance mixing, generate zero order mixing pattern, produce randomness (if needed). Flavor symmetries at high (GUT, above GUT) scales can ensure specific form of the RH neutrino mass matrix, and consequently, specific form of m_ν.

5. – Mixing: symmetry or no symmetry

The observed pattern of the lepton mixing can be described by the approximate TriBimaximal (TBM) mixing [25]. Is a symmetry behind the mixing (usually non-abelian discrete symmetries are used [26]) real or accidental? TBM looks strange in a sense that it is difficult, if possible, to connect it to the lepton masses although both masses and mixing result from diagonalization of the same mass matrices. A framework which could realize such a feature is that mixing originates from different ways of the flavor symmetry breaking in the neutrino and charged lepton (Yukawa) sectors [27]. These different ways lead to different residual symmetries of the mass matrices of neutrinos and charged leptons:

$$G_f \rightarrow \begin{cases} G_\nu \\ G_l \end{cases} . \tag{10}$$

Furthermore, G_ν and G_l should be generic symmetries which do not depend on the values of masses. This ensures maximal control of mixing by symmetries.

It has been shown that in this framework the mixing parameters or relations between them can be obtained without model-building, immediately from knowledge of the residual symmetries [28, 9]. Model-building is the ugliest part of the construction: it requires many assumptions, *ad hoc* introduction of new fields, auxiliary symmetries, new tuned parameters. One however can skip this "unpleasant" part and obtain final results in one step even without construction of mass matrices:

(11) *residual symmetries* $\rightarrow$ *relations between mixing parameters.*

This can be done using the symmetry group condition [28]:

$$(U_{PMNS} S_i U_{PMNS}^\dagger T)^p = I. \tag{12}$$

Here S_i and T are the symmetry transformations of the diagonal neutrino and charged lepton mass matrices (*i.e.* matrices in the mass bases) and p is an integer. In the flavor basis (T is kept the same) the transformation matrix of neutrinos becomes $S_{iU} = U_{PMNS} S_i U_{PMNS}^\dagger$. So, the relation (12), which can be rewritten as $(S_{iU}T)^p = I$, is nothing but the condition that the elements S_{iU} and T form a finite group. (The product $W \equiv S_{iU}T$ also belongs to the group and due to finiteness the integer p exists such that $W_i^p = I$) ([1]).

The relation (12) connects the mixing matrix U_{PMNS} and generating elements of the group in the mass bases. It is equivalent to

$$\mathrm{Tr}(U_{PMNS} S_i U_{PMNS}^\dagger T) = a, \quad a = \sum_j \lambda_j, \quad \lambda_j^p = 1, \tag{13}$$

and λ_j are three eigenvalues of W_i [28]. The generic (mass independent) transformation matrices for neutrinos are $S_1 = diag(1, -1, -1)$ and $S_2 = diag(-1, 1, -1)$ which correspond to maximal generic symmetry $\mathbf{Z}_2 \times \mathbf{Z}_2$, and for charged leptons one can take $T = diag\left(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau}\right)$, where $\phi_\alpha = 2\pi\kappa_\alpha/m$ corresponds to $\mathbf{Z}_m$.

For single fixed S_i ($G_\nu = \mathbf{Z}_2$) the relation (13) determines ith column of the mixing matrix. Maximal $G_\nu = \mathbf{Z}_2 \times \mathbf{Z}_2$ fixes two columns and therefore determines the mixing matrix completely. The only phenomenologically viable structure here is TBM and so it seems that indeed TBM is special. One can scan all the possibilities varying p, κ_α, m reproducing results of [29]. However, only for special sets of these parameters the group is finite, see [30].

([1]) Notice that the whole information about mixing is in the mass matrices only in the basis where the charged currents are diagonal. So, the neutrino and charged lepton symmetry transformations should be taken in such a basis.

The symmetry G_ν can be extended by transformations which leave the neutrino mass matrix invariant only for specific mass spectra [31]. In this case relations will include also masses and Majorana CP phases. For unitary symmetry transformations the possibilities include a) Partially degenerate spectrum, $m_1 = m_2,\ m_3$, and $S_\nu = O(2)$, so that $G_\nu = SO(2) \times Z_2$. That leads to the relations [31]

$$\sin^2 2\theta_{23} = \pm \sin\delta = \cos\kappa = m_1/m_2 = 1 \tag{14}$$

with 1-2 mixing being undefined. The relation (14) can be a good first order approximation both for normal and inverted mass hierarchies. Relatively small corrections to the mass matrix can lead to the required 1-2 mass splitting and mixing. b) Degenerate mass spectrum $m_1 = m_2 = m_3$ [31]. c). Spectrum with one zero mass $m_1 = 0, m_2, m_3$, in which case G_ν is a subgroup of $U(3)$ [32].

Other scenario is when flavor symmetry is broken down to the same residual symmetry in neutrino and lepton sectors or no symmetry is left. (The same flavon fields are responsible for the neutrino and charged lepton mass generation.) In this case mixing can originate from (i) different nature of the mass terms of the charged leptons (Dirac) and neutrinos (Majorana), (ii) mixing of neutrinos with new degrees of freedom S - singlets of SM.

6. – Remark on sterile neutrinos

I am sure Bruno Maksimovich would enjoy knowing that a number of researchers working on his sterile neutrinos got emails saying "Dear Dr. ... , Please pay attention to our upcoming Special Issue on "Research in Sterility" which will be published in the "Advances in Sexual Medicine", an open access journal. We cordially invite you to submit your paper."

"Steriles" with mixing parameters, U_{iS}, and mass m_S required by the LSND/MiniBooNE, reactor and gallium anomalies [33] are a non small perturbation of the 3ν picture. In the presence of steriles the mass matrix of active neutrinos becomes [34]

$$m_\nu = m_a + \delta m, \tag{15}$$

where m_a is the original active neutrino mass matrix which follows, e.g., from see-saw, and δm is the mass matrix induced by mixing with steriles. In the $(3+1)$ scheme $\delta m_{ij} \approx -U_{iS} U_{jS} m_S \approx 0.04 m_S$. That is, $\delta m \approx 0.04$ eV if $m_S \sim 1$ eV, which is comparable with the largest elements of m_a for hierarchical mass spectrum. So, the correction δm is not a perturbation, it can change structure (symmetries) of the original mass matrix completely: it can produce the dominant $\mu\tau$ - block with small determinant, enhance lepton mixing, generate TBM mixing, be, in general, the origin of the difference between U_{PMNS} and V_{CKM}. Thus, checks of existence of these steriles are of the highest priority for further theoretical advances.

7. – Conclusions and outlook

A theory of neutrino masses and mixing is still at the cross-roads with many possibilities. In particular, the same value of 1-3 mixing satisfies various relations which have different implications. Interesting possibilities include natural mass mixing relation, the

QLC relation, special violation of the $\nu_\mu - \nu_\tau$ symmetry, quark-lepton similarity, *etc.*. The question "Can we go beyond D5 Weinberg's operator?" is still open.

Elements of theory which have some chance to reflect reality probably include connection between zero charges, smallness of mass and large mixing, unified description of quarks and leptons, existence of the RH neutrinos without special quantum numbers.

Low scale mechanisms of ν mass generation are not much along the guidelines, but they are testable and deprived of the hierarchy problem. In view of data from LHC, MEG, etc., the minimalistic phenomenological scenario of νMSM looks more plausible than before. Still a scenario with high scale seesaw, probably in some extended version (with more RH neutrinos involved), some flavor symmetry at the high mass scale, unification of quarks and leptons, similarity of the Dirac structures in both sectors looks appealing. The high scale seesaw creates the hierarchy problem and influences stability of the Higgs potential. Solutions of these problems may lead to some new developments.

It looks like the RH neutrinos their existence or non-existence, their number and properties are the key to understand mass and mixing of light neutrinos. Smallness of neutrino mass may be connected to other hierarchies.

Concerning mixing pattern, the main issue is "Symmetry or no symmetry" behind the observed pattern. TBM could be accidental and symmetry behind - misleading in searches for underlying physics. As the zero order structure it is still possible, and still useful as book-keeping for phenomenological considerations.

The symmetry group relations are a powerful tool for studies of consequences of discrete flavor symmetries for lepton mixing and masses. They are useful for "symmetry building": uncovering symmetry for a given mixing pattern.

Sterile neutrinos are a challenge for the standard 3ν scenario. Tests of the existence of these sterile neutrinos are of paramount importance.

REFERENCES

[1] G. Altarelli and F. Feruglio, New J. Phys. **6** (2004) 106; R. N. Mohapatra and A. Y. Smirnov, Ann. Rev. Nucl. Part. Sci. **56** (2006) 569; G. Altarelli and F. Feruglio, Rev. Mod. Phys. **82** (2010) 2701 S. F. King and C. Luhn, Rept. Prog. Phys. **76** (2013) 056201.

[2] A. de Gouvea and H. Murayama, Phys. Lett. B **573** (2003) 94; arXiv:1204.1249 [hep-ph].

[3] F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino and A. Palazzo, arXiv:1312.2878 [hep-ph].

[4] E. K. Akhmedov, G. C. Branco and M. N. Rebelo, Phys. Rev. Lett. **84** (2000) 3535; W. Rodejohann, M. Tanimoto and A. Watanabe, Phys. Lett. B **710** (2012) 636.

[5] C. Giunti and M. Tanimoto, Phys. Rev. D **66** (2002) 113006.

[6] H. Minakata and A. Y. Smirnov, Phys. Rev. D **70** (2004) 073009.

[7] M. Raidal, Phys. Rev. Lett. **93** (2004) 161801.

[8] D. A. Eby and P. H. Frampton, Phys. Rev. D **86** (2012) 117304.

[9] A. Y. Smirnov, J. Phys. Conf. Ser. **447** (2013) 012004 [arXiv:1305.4827].

[10] S. Weinberg, Phys. Rev. Lett. **43** (1979) 1566.

[11] P. Minkowski, Phys. Lett. B **67** (1977) 421. T. Yanagida, in Proc. of Workshop on Unified Theory and Baryon number in the Universe, eds. O. Sawada and A. Sugamoto, KEK, Tsukuba, (1979); M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Niewenhuizen and D. Z. Freedman (North Holland, Amsterdam 1980); S. L. Glashow, in Quarks and Leptons, Cargese lectures, eds M. Levy, (Plenum, 1980, New York) p . 707; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, (1980) 912.

[12] F. Vissani, Phys. Rev. D **57** (1998) 7027; M. Farina, D. Pappadopulo and A. Strumia, JHEP **1308** (2013) 022.

[13] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto and A. Strumia, Phys. Lett. B **709** (2012) 222.
[14] D. Wyler and L. Wolfenstein, Nucl. Phys. B **218** (1983) 205. R. N. Mohapatra, Phys. Rev. Lett. **56** (1986) 561. R. N. Mohapatra and J. F. W. Valle, Phys. Rev. D **34** (1986) 1642.
[15] P. S. B. Dev and A. Pilaftsis, Phys. Rev. D **86** (2012) 113001.
[16] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali and J. March-Russell, Phys. Rev. D **65** (2002) 024032.
[17] Y. Grossman and M. Neubert, Phys. Lett. B **474** (2000) 361.
[18] E. Ma, Phys. Rev. D **73** (2006) 077301 [hep-ph/0601225].
[19] K. S. Babu, Phys. Lett. B **203** (1988) 132, A. Zee, Phys. Lett. B **93** (1980) 389 [Erratum-ibid. B **95** (1980) 461]. K. S. Babu, A. Patra and S. K. Rai, Phys. Rev. D **88**, 055006 (2013).
[20] A. Maiezza, M. Nemevsek, F. Nesti and G. Senjanovic, Phys. Rev. D **82** (2010) 055022; M. Nemevsek, et al., Phys. Rev. D **83** (2011) 115014; P. S. B. Dev and R. N. Mohapatra, arXiv:1308.2151 [hep-ph].
[21] W. -Y. Keung and G. Senjanovic, Phys. Rev. Lett. **50** (1983) 1427.
[22] V. Tello, et al, Phys. Rev. Lett. **106** (2011) 151801, P. S. Bhupal Dev, et al, 1305.0056 [hep-ph].
[23] T. Asaka, S. Blanchet and M. Shaposhnikov, Phys. Lett. B **631** (2005) 151, L. Canetti, et al., Phys. Rev. D **87** (2013) 093006.
[24] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659** (2008) 703.
[25] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530** (2002) 167, L. Wolfenstein, Phys. Rev. D **18** (1978) 958.
[26] S. Pakvasa and H. Sugawara, Phys. Lett. B **73** (1978) 61. G. C. Branco, Phys. Lett. B **76** (1978) 70, E. Ma and G. Rajasekaran, Phys. Rev. D **64** (2001) 113012.
[27] C. S. Lam, Phys. Rev. D **78** (2008) 073015, Phys. Rev. Lett. **101** (2008) 121602, C. S. Lam, Phys. Rev. D **87** (2013) 013001, W. Grimus, L. Lavoura and P. O. Ludl, J. Phys. G **36** (2009) 115007, W. Grimus and L. Lavoura, JHEP **0904** (2009) 013.
[28] D. Hernandez and A. Y. Smirnov, Phys. Rev. D **86** (2012) 053014; Phys. Rev. D **87** (2013) 5, 053005.
[29] M. Holthausen, et al., Phys. Lett. B **721** (2013) 61.
[30] C. S. Lam, Phys. Rev. D **87** (2013) 053018.
[31] D. Hernandez and A. Y. Smirnov, Phys. Rev. D **88** (2013) 093007.
[32] A. S. Joshipura and K. M. Patel, Phys. Lett. B **727** (2013) 480.
[33] C. Giunti, *these proceedings*.
[34] A. Y. Smirnov and R. Zukanovich Funchal, Phys. Rev. D **74** (2006) 013001.