Phenomenology of dihadron FF: Collinear extraction of the valence transversities

A. Courtoy(1), Alessandro Bacchetta(2)(4) and Marco Radici(3)

(1) IFPA, AGO Department, Université de Liège - Bât. B5, Sart Tilman B-4000 Liège, Belgium
INFN, Laboratori Nazionali di Frascati - Via E. Fermi, 40, I-00044 Frascati (RM) Italy
(2) Dipartimento di Fisica, Università di Pavia - via Bassi 6, I-27100 Pavia, Italy
(3) INFN, Sezione di Pavia - via Bassi 6, I-27100 Pavia, Italy

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Summary. — In this paper, we propose an extraction of the valence transversity parton distributions. Based on an analysis of pion-pair production in deep-inelastic scattering off transversely polarized targets, this extraction of transversity is performed in the framework of collinear factorization. The recently released data for proton and deuteron targets at HERMES and COMPASS allow for a flavor separation of the valence transversities, for which we give a complete statistical study.

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1. – Introduction

The transversity parton distribution function (PDF) is poorly known with respect to the 2 other leading-twist distributions. In consequence of its chiral-odd nature, transversity is not observable from fully inclusive Deep Inelastic Scattering (DIS). It can be measured in processes that either have two hadrons in the initial state, e.g., proton-proton collision; or one hadron in the initial state and at least one hadron in the final state, e.g., semi-inclusive DIS (SIDIS).

It was considering the latter kind of processes —more precisely, single-hadron SIDIS—that the valence transversities were extracted for the very first time by the Torino group [1]. The main shortcoming in analyzing such processes, and the deriving outcomes, lies in the factorization framework they must obey. In effect, most single-hadron SIDIS processes involve Transverse Momentum Dependent functions (TMDs), for which the community fails in coming together to an unique definition. An alternative to this issue arose thanks to the so-called Dihadron Fragmentation Functions (DiFFs) [2,3], that are defined in a collinear factorization framework.
In this contribution to the proceedings, we give a parameterization of the valence transversities together with the error analyses [4]. The fitting procedure is based on the knowledge of DiFFs from a previous analysis [5]. The outcome confirms our first extraction of the proton transversity [6].

2. Two-hadron semi-inclusive DIS

We consider two-hadron production in DIS, \( \ell (l) + N(P) \rightarrow \ell (l') + H_1(P_1) + H_2(P_2) + X \) where \( \ell \) denotes the beam lepton, \( N \) the nucleon target, \( H_1 \) and \( H_2 \) the produced hadrons, and where four-momenta are given in parentheses. For a transversely polarized target, the cross section is related to the simple product of the (collinear) transversity distribution function and the chiral-odd DiFF, denoted as \( H_1^{q} \) [3]. The latter describes the correlation between the transverse polarization of the fragmenting quark with flavor \( q \), related to \( \phi_S \), and the azimuthal orientation of the plane containing the momenta of the detected hadron pair, related to \( \phi_R \). In such a process, the “transverse” reference comes from the projection of the relative momentum of the hadron pair onto the plane transverse to the jet axis. This leads to an asymmetry in \( \sin(\phi_S + \phi_R) \).

Hence, in such a collinear framework, we can single out the DiFF contribution to the SIDIS asymmetry from the \( x \)-dependence coming from the transversity PDF. In particular, the \( x \) behavior of \( h_1(x) \) is simply given by integrating the numerator of the asymmetry over the \( (z,M_h) \)-dependence. This procedure then defines the quantities \( n_q \) and \( n_{\uparrow q} \) that are the integrals of, respectively, the unpolarized and chiral-odd DiFFs. The relevant single-spin asymmetry in SIDIS here can be expressed in terms of the integrated DiFFs. It reads

\[
A_{\text{SIDIS}}(x,Q^2) = -C_y \sum_q e_q^2 h_1^q(x,Q^2)n_{\uparrow q}(Q^2)
\]

where the sum runs over all the quark flavors. The depolarization factor \( C_y = 1 \) for COMPASS data and \( C_y \approx (1-\langle y \rangle)/(1-\langle y \rangle +\langle y^2 \rangle/2) \) for HERMES data.

Given the asymmetry data and using \( f_1(x) \) from available sets, the remaining unknowns in eq. (1) are the transversities and the integral of the DiFFs. The latter are extensively described in refs. [4,5]. We here only need to recall that we used isospin symmetry and charge conjugation to relate the polarized or unpolarized DiFFs of different flavors [7], what leaves only \( n_u, n_s \) and \( n_{\uparrow u} \) to the purpose.

3. The extraction from a collinear framework

Starting with the expression (1) for the asymmetry, we discuss the extraction of the valence transversities from combining data on both proton and deuteron target from COMPASS and proton target from HERMES. The main theoretical constraint we have is Soffer’s inequality [8],

\[
2|h_1^q(x;Q^2)| \leq |f_1^q(x;Q^2)| + |g_1^q(x;Q^2)| \equiv 2SB^q(x;Q^2).
\]

If the Soffer bound is fulfilled at some initial \( Q_0^2 \), it will hold also at higher \( Q^2 \geq Q_0^2 \). We impose this positivity bound by multiplying the functional form by the corresponding Soffer bound at the starting scale of the parameterization, \( Q_0^2 = 1 \) GeV\(^2\). The implementation of the Soffer bound depends on the choice of the unpolarized and helicity PDFs.
The valence transversities at \( Q^2 = 2.4 \text{ GeV}^2 \). The thick solid blue lines are the Soffer bound. The uncertainty band (red stripes) with solid boundaries is the best fit in the standard approach at \( \sigma \), whose central value is given by the central thick solid red line. The uncertainty band (plain green) with dashed boundaries is the 68\% of all fitting replicas obtained in the Monte Carlo approach. As a comparison, the uncertainty band (blue stripes) with short-dashed boundaries is the transversity extraction from the Collins effect [1].

We use the MSTW08 set [9] for the unpolarized PDF, combined to the DSSV parameterization [10] for the helicity distribution, at the scale of \( Q^2_0 = 1 \text{ GeV}^2 \). Our analysis was carried out at LO in \( \alpha_S \). The functional form that ensues for the valence transversity distributions reads

\[
x h_V^q(x; Q^2_0) = \tanh \left[ x^{1/2} \left( A_q + B_q x + C_q x^2 + D_q x^3 \right) \right] \\
\times \left[ x SB^q(x; Q^2_0) + x SB^q(x; Q^2_0) \right].
\]

The hyperbolic tangent is such that the Soffer bound is always fulfilled. The functional form is very flexible and can contain up to three nodes but the low-\( x \) behavior is determined by the \( x^{1/2} \) term, which is imposed by hand in order to insure the integrability of the transversity PDF. We have considered different sets of parameters. However, in these proceedings, we will give results for the set which contains 6 parameters, i.e., \( D_u = D_d = 0 \). We call it the flexible scenario.

The fit, and in particular the error analysis, was carried out in two different ways: using the standard Hessian method and using a Monte Carlo approach. The latter is inspired from the work of the NNPDF collaboration, e.g., [11], although our results are not based on a neural-network fit. The approach consists in creating \( N \) replicas of the data points, shifted by a Gaussian noise with the same variance as the measurement. Each replica, therefore, represents a possible outcome of an independent experimental measurement. Then, the standard minimization procedure is applied to each replica separately as explained in ref. [4].

We show the resulting functional form for the valence transversities at \( Q^2 = 2.4 \text{ GeV}^2 \) of both fitting procedures for the flexible scenarios on fig. 1. In the standard Hessian method, the \( \chi^2/\text{d.o.f.} \) is 1.12 for the flexible scenario, while the average \( \chi^2 \) is \( \chi^2/\text{d.o.f.} = 1.56 \) for 100 replicas.

The uncertainty bands in the standard and Monte Carlo approaches are quite similar. The main difference is that in the former case the boundaries of the band can occasionally cross the Soffer bound. This is due to the fact that the hypothesis under which we can use the standard Hessian approach are no longer valid in this limit. On the contrary, in the
Monte Carlo approach each replica is built such that it never violates the Soffer bound; the resulting 68% band is always within those limits. We observe that the standard approach for the $u\nu_t$ tends to saturate the Soffer bound at $x \sim 0.4$. In the Monte Carlo approach, some of the replicas saturate the bound already at lower values. This happens in both approaches at values of $x$ for which no data are available. In the range where data exist, our results are compatible with the only other existing parametrization of transversity [1]. The only source of discrepancy lies in the range $0.1 < x < 0.16$ for the valence down quark\(^{(1)}\).

A qualitative comparison between our results and the available model predictions shows that the extracted transversity for the up quark is smaller than most of the model calculations at intermediate $x$, while it is larger at lower $x$. The down transversity is much larger in absolute value than all model calculations at intermediate $x$ (as observed before, this is due to the deuterium data points), while the error band is too large to draw any conclusion at lower $x$.

4. – Conclusions

We have proposed a parametrization of the valence transversities in a collinear framework. It relies on the knowledge of DiFFs, which we have studied and parametrized in previous works. The transversity analysis is driven in two different statistical approaches: a standard and a Monte-Carlo–like one.

We can conclude that, outside the kinematical range of experiments, the lack of data reflects itself in a large uncertainty in the parametrization. This illustrates the need for new large-$x$ data in order to reduce the degree of uncertainty in the knowledge of transversity. Also, since the tensor charge is defined as the integral of the transversity PDF on its support, the shape of the valence transversities, at both low and large-$x$, plays a key role in determining the tensor charge. Data from JLab (expected in the next years) should help unfolding the situation.

REFERENCES


\(^{(1)}\) It might be due to two experimental data for the deuteron target.