

Quantum games with a multi-slit electron diffraction set-up^(*)

A. IQBAL^(**)

*HuMP, Hull Mathematical Physics, Department of Mathematics, University of Hull
Cottingham Road, Hull HU6 7RX, United Kingdom*

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Summary. — A set-up is proposed to play a quantum version of the famous bimatrix game of Prisoners' Dilemma. Multi-slit electron diffraction with each player's pure strategy consisting of opening one of the two slits at his/her disposal are essential features of the set-up. Instead of entanglement the association of waves with travelling material objects is suggested as another resource to play quantum games.

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1. – Introduction

Quantum games has become the subject of many recent interesting papers [1-7]. Meyer, in a seminal work [1], presented the idea of playing a sequential quantum game with unitary manipulation of a qubit. Shortly afterwards Eisert, Wilkens, and Lewenstein [2, 4] aroused a lot of interest in quantum games by showing that the dilemma in the famous classical game of Prisoners' Dilemma (PD) can disappear when the game is played with quantum-mechanical maneuvers consisting of local and unitary manipulation of a pair of maximally entangled qubits. Much of the later analysis of quantum games consists of exploiting the popular resource of quantum information, *i.e.* the entanglement, in many different proposed set-ups. Many authors have made their valuable contributions by providing many interesting scenarios for quantum games [8-13]. Unitary manipulation of entangled systems of qubits is one of the favorite general concepts to play a quantum game [1-3, 5-7]. Though it is shown that entanglement may not be essential for a quantum game [14] but it continues to play its role as a resource. A noticeably greater attention paid to many suggestions exploiting entanglement for quantum games can be

^(*) The author of this paper has agreed to not receive the proofs for correction.

^(**) E-mail: a.iqbal@maths.hull.ac.uk

traced back to the exciting and counter-intuitive properties of this phenomenon, as well as to its recent active and detailed investigation in quantum information theory [15].

Unitary manipulation of entangled qubits to play matrix games is indeed an interesting concept giving new dimensions to the classical game theory. But it does not forbid the use of other quantum-mechanical effects to play other “quantum forms” of the matrix games—games for which extensive classical analysis is already found in the literature [16,17]. A look at Eisert *et al.*'s idea [2,4] makes apparent some of its similarities to the well-known Young double-slit apparatus [18]. Simultaneous and local unitary manipulation of a maximally entangled two-qubit quantum state and its later measurement is the essential feature of Eisert *et al.*'s idea. In Young's double-slit case, however, a coherent light passes through two slits to form a diffraction pattern on a screen facing the slits. Similarity between these set-ups becomes noticeable if one draws a kind of comparison between the properties of entanglement and coherence, players' moves and the process of opening or closing the slits, wave-function-collapsing measurement and the appearance of the diffraction pattern, in the set-ups of Eisert *et al.* and Young, respectively. This comparison in its turn asks for some quantum feature that can be exploited to give some new dimension to a matrix game when played using a Young double-slit-like apparatus. In Eisert *et al.*'s set-up this quantum feature is obviously the entanglement. In Young's apparatus this feature can be, for example, the association of well-known wave-like properties to material objects like electrons, producing a diffraction pattern on a screen.

Historically speaking De Broglie's original idea [18,19], that travelling material particles have waves associated with them, was undoubtedly the key concept behind the development of quantum physics in early part of the twentieth century. Soon afterwards Davisson and Germer [18] verified the idea in their experimental demonstration of the diffraction of electrons by crystals. In De Broglie's argument a travelling electron with momentum p has an associated wave of wavelength $\lambda = h/p$, where h is Plank's constant. To make λ a measurable quantity, under normal laboratory conditions, the momentum p should have similar order of magnitude as h . It shows why it is very hard to detect waves associated with macroscopic objects. Our motivation is to take the quantum feature, that associates wave-like properties with micro objects, as a resource that can be used to play a quantum game. Such a quantum game can be realized using an apparatus consisting of travelling electrons, multiple slits intercepting them, and a resulting diffraction pattern. In this set-up a player's choice of a “pure strategy” will consist of opening or closing slits at his/her disposal. Suppose the apparatus is adjusted such that when λ approaches zero the classical game is reproduced. It can then be argued that because an observation of a value of λ quite away from zero is entirely a quantum feature, therefore, the resulting different payoffs for the players correspond to a quantum form of the classical game. In this set-up the players' payoffs are to be found from the diffraction pattern formed on the screen. We show the possibility of finding a value for λ that makes a non-classical equilibrium in the PD game appear when the players play only the pure strategies. The classical game remains a subset of its quantum version because with λ approaching zero the classical game is reproduced.

2. – Playing quantum games with a diffraction apparatus

Eisert *et al.*'s first investigation of quantum Prisoners' Dilemma (PD) [2,4] has provided much of the later motivation for a systematic study of quantum games. The classical game of PD, in its general form, can be represented by the matrix in fig. 1.

		Bob		C: cooperate D: defect
		C	D	
Alice	C	(r,r)	(s,t)	r: reward p: punishment
	D	(t,s)	(p,p)	t: temptation s: sucker's payoff

Fig. 1. – Payoff matrix for general Prisoners’ Dilemma. The first and the second entries in a parenthesis are Alice’s and Bob’s payoffs, respectively. For Prisoners’ Dilemma the condition $t > r > p > s$ should hold (see in ref. [20,21]).

Our motivation to play a quantum form of PD game, without using the phenomenon of entanglement, derives from Feynman’s excellent exposition [22] of quantum behavior of atomic objects. He describes and compares the diffraction patterns—in two similar imaginary, but experimentally realizable, set-ups—consisting of bullets and electrons passing through a wall with two slits. Feynman describes a well-known quantum property—associating waves to all material particles—to distinguish the diffraction patterns of bullets and electrons. The disappearance of a pattern for the bullets is then explained as due to tiny wavelengths of the associated waves, such that the pattern becomes very fine, and with a detector of finite size one cannot distinguish the separate maxima and minima. We asked why not to play a game, in Feynman’s imaginary experimental set-up, such that one gets the classical game when, in Feynman’s words, bullets are fired and a quantum game corresponds when electrons replace the bullets. The experimental set-up shown in fig. 2 illustrates the point.

We now select the famous classical PD game to be played in this set-up. To make the classical game imbedded in its quantum version the positive coefficients p, r, s , and

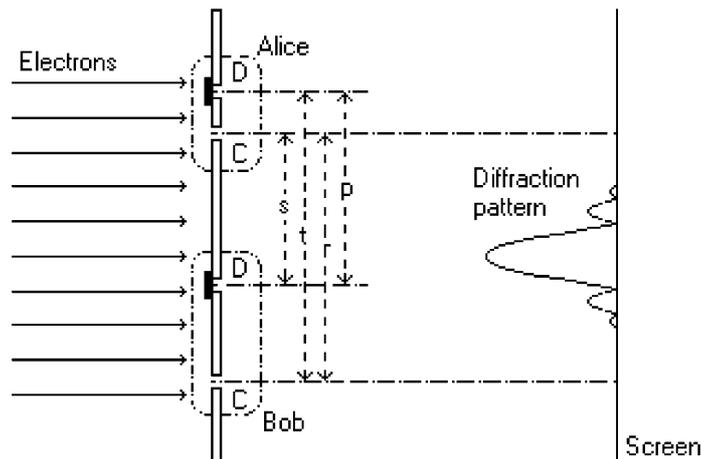


Fig. 2. – A multi-slit diffraction set-up to play a quantum form of Prisoners’ Dilemma. A window with four slits faces an electron source. Each player has access to two slits. A player plays a pure strategy by opening a slit and closing the other. Arbitrarily finds the players’ payoff by measuring the peak-to-peak distance on the diffraction pattern formed on the screen.

t , appearing in the matrix representation of the game, are translated into the distances between the slits. Each player is in control of two slits such that his/her strategy consists of opening one of the slits and closing the other. For example, if Alice decides to cooperate then she would open the slit C and close D . Because Bob has a similar choice, therefore, every possible move by the players leads to opening of two slits and closure of the other two, with the separation between the two open slits depending on the moves of the players. It will happen when only the so-called “pure-strategies” can be played by the players, which in the present set-up means to open a slit and close the other. Now, at the final stage of the game, the action of the arbiter—who is responsible for computing the payoffs when the players have made their moves—consists of measuring the distance between two peaks of the diffraction pattern. This peak-to-peak distance is known to be λ/d [18], where d is the separation between the open slits and λ is the wavelength associated with the bombarded material objects, like electrons. The payoffs for the players are the functions of λ/d and it, then, explains the utility of translating the coefficients of the matrix of the classical game into separations d between the slits. When bullets are fired, which means here the particles become heavier and corresponding λ is very nearly zero, the payoffs become classical and depend only on d , *i.e.* the separation between the slits. A payoff representation in terms of λ/d contains both the classical and quantum aspects of the matrix game played in this set-up.

For PD the payoffs are symmetric for the players and a single equation can describe the payoffs to both the players when their strategies are known. A usual way to express it is to write $P(s_1, s_2)$ for the payoff to the s_1 -player against the s_2 -player. Such a single equation representation is usually used in evolutionary games [23] consisting of symmetric bimatrix conflicts. The s_1 -player is referred to as the “focal” player and the s_2 -player as just the “other” player. The PD is one such example for which a single payoff equation can capture the essence of the idea of a symmetric Nash Equilibrium (NE). The strategy s^* is symmetric NE if

$$(1) \quad P(s^*, s^*) - P(s, s^*) \geq 0, \quad \text{for all } s \neq s^*$$

saying that the focal player cannot be better off by diverging away from s^* . Because the set-up of fig. 2 involves only the coefficients in the classical payoff matrix corresponding to the first player, therefore, finding a symmetric NE with eq. (1) becomes possible immediately when the first player is taken as focal. It also shows why writing payoff as $P(s_1, s_2)$ is relevant to the set-up of fig. 2. For example, classically the strategy of defection D comes out as a symmetric NE because $P(D, D) - P(C, D) = (p - s) > 0$, when the players’ moves consist of the pure strategies only.

In the set-up of fig. 2, for every pure strategy move the players have option to make, a unique separation d between the slits is obtained that can have four possible values, *i.e.* p, r, s or t . Classically $P(C, C) = r$, $P(C, D) = s$, $P(D, C) = t$, and $P(D, D) = p$. It can be noticed in fig. 2 that the classical payoff to the focal player against the other can be equated to the separation between the two open slits d .

Now assume that the arbiter uses the following payoff equation, instead of simply $P(s_1, s_2) = d$:

$$(2) \quad P(s_1, s_2) = d + k(\lambda/d),$$

where k is a positive constant that can be called a scaling factor. $P(s_1, s_2)$, obviously, reduces to its classical counterpart when λ is very nearly zero. Suppose the strategy of

cooperation C is a symmetric NE, then

$$(3) \quad P(C, C) - P(D, C) = \{k\lambda(1/r - 1/t) - (t - r)\} \geq 0.$$

It requires $\lambda \geq rt/k$. For electrons of mass m travelling with velocity v it gives $v \leq (kh/mrt)$. Supposing r and t are both non-zero, the arbiter's problem consists of finding an appropriate value for the scaling factor k for which v comes in a reasonable range from the experimental point of view. When the electrons have a $\lambda \geq rt/k$ the strategy of cooperation becomes a symmetric NE and each player gets a payoff $r + k\lambda/r$. Similarly when the pure strategy of defection is a symmetric NE in the quantum game

$$(4) \quad P(D, D) - P(C, D) = \{-k\lambda(1/s - 1/p) + (p - s)\} \geq 0.$$

It requires $\lambda \leq sp/k$. After the scaling factor k is determined the wavelength λ decides which pure strategy should be a symmetric NE. Two ranges for λ are, therefore, indicated, *i.e.* $\lambda \leq sp/k$ and $\lambda \geq rt/k$. Defection and cooperation come up as symmetric NE for these ranges, respectively. Because the constants t, r, p, s and k are all positive the classical game is in the earlier range of λ . Non-classical equilibrium of cooperation shows itself in the later range of λ .

Du *et al.*'s recent analysis [6] of the quantum PD, with players' access to Eisert's set of two-parameter set of unitary operators, has shown an intriguing structure in the game as a function of the amount of entanglement. The game turns into classical with the amount of entanglement becoming zero. In the set-up of fig. 2 the quantity λ behaves in similar way like the amount of entanglement in Du *et al.*'s analysis [6, 20]. But the present set-up, to play a quantum game, is devoid of the notion of entanglement and relies instead on a very different quantum aspect, which is as much quantum in nature as the phenomenon of entanglement for qubit systems. There is however a difference, to be noticed, between the set-ups of Eisert *et al.* and of fig. 2. Players' actions in Eisert *et al.*'s set-up are very much quantum mechanical in nature in that they make moves with operators from the quantum world. In the present set-up, on the contrary, the players' actions are entirely classical consisting of opening or closing slits. It is similar to the players' actions in Marinatto and Weber's idea [5] of playing a quantum form of the matrix game of the battle of sexes. In this scheme players possess quantum operators but they apply those on an initial quantum state with classical probabilities, so that the players' moves can be considered classical as well. A transition to the classical game is obtained by unentangling the initial state. It can be observed that, apart from the pioneering work of Eisert *et al.*, the set-up of fig. 2 is also motivated, to an almost equal extent, by Marinatto and Weber's idea [5] of playing a quantum version of a matrix game.

3. – Concluding remarks

In many of the earlier suggested set-ups some appropriate measure of entanglement for a qubit system is introduced. The quantum version of the game reduces to classical when the measure becomes zero. Instead of the entanglement we introduce another resource from quantum physics, *i.e.* the association of waves with travelling material objects like electrons, to show how it can lead to a non-classical equilibrium in the PD game. With associating wavelength approaching zero the quantum aspect disappears and the classical game is reproduced. Like entanglement the association of waves with particles is a purely

quantum phenomenon rightly considered a cornerstone of the quantum theory, though less esoteric because of its earlier discovery. We suggest its exploitation as another resource to play quantum games.

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