Black-hole creation in quantum cosmology

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Summary. — It is proven that the probability of a black hole created from the de Sitter space-time background, at the WKB level, is the exponential of one quarter of the sum of the black hole and cosmological horizon areas, or the total entropy of the universe. This is true not only for the spherically symmetric cases of the Schwarzschild or Reissner-Nordström black holes, but also for the rotating case of the Kerr black hole and the rotating charged case of the Newman black hole. The de Sitter metric is the most probable evolution at the Planckian era of the universe.

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1. - Introduction

It is believed that the very early universe is approximately described by a de Sitter metric. In quantum cosmology, at the Planckian era, the universe was created from a $S^4$ space through a quantum transition. Therefore, to study the problem of primordial black-hole creation in the de Sitter space-time background is of twofold interest, for cosmology and for black-hole physics.

There has been some progress in this direction. However, nearly all scenarios studied are associated with pair creation of black holes [1-5]. The main reason for this is that people consider our universe to have been created by a quantum transition from a gravitational instanton. There does not exist any gravitational instanton which provides the seed for the creation of a single black hole in the de Sitter background.

A gravitational instanton is a regular Euclidean solution to the Einstein equation.

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Only very recently has it been realized [6, 7] that for many reasons one should consider the so-called generalized gravitational instanton in quantum cosmology.

The space of metrics over which the path integral for the universe ground state of Hartle and Hawking is summed [8] should be extended, in order for the theory to be consistent. As we mentioned above, a gravitational instanton leads to a Lorentzian trajectory. The probability of a Lorentzian evolution emanating from a 3-surface $\Sigma$ with matter fields $\phi$ on it can be written as

$$ P = \Psi^* \Psi = \int_C d[g_{\mu\nu}] d[\phi] \exp \left[ - \tilde{I}([g_{\mu\nu}, \phi]) \right], $$

where the path integral is over class $C$ of all no-boundary compact Euclidean 4-metrics and matter field configurations which agree with the given 3-metric $h_{ij}$ and matter field $\phi$ on the equator $\Sigma$. $\tilde{I}$ is the Euclidean action.

The Euclidean action for the gravitational part of a smooth space-time manifold $M$ with boundary $\partial M$ is

$$ \tilde{I} = \frac{1}{16\pi} \int_M (R - 2\Lambda) - \frac{1}{8\pi} \int_{\partial M} K, $$

where $R$ is the scalar curvature, $K$ is the trace of the second fundamental form of the boundary.

When we derive formula (1) from the no-boundary proposal of Hartle and Hawking, we have implicitly introduced some jump discontinuities of the extrinsic curvature at the 3-geometry $\Sigma$. Therefore, one has to allow the manifold $M$ at any location to contain these kinds of mild singularities. The degenerate case is the conical or pancake singularity. The main contribution to the path integral in eq. (1) is due to the stationary action 4-metric, which meets all requirements on the 3-surface $\Sigma$ and other restrictions. At the WKB level the exponential of the negative of the lowest stationary action is the probability of the corresponding Lorentzian trajectory. Furthermore, in real time it corresponds to the most probable Lorentzian trajectory. Under this scheme quantum cosmology realizes its complete predictive power. Indeed, no degree of freedom is then left except for a physical time [9]. If, in some sense, class $C$ is not complete, then one may not be able to find the most probable trajectory.

If a 3-surface $\Sigma$ is not the equator of an instanton, then the Lorentzian evolution emanating from it is not the most probable one. In order to find its probability at the WKB level, one still can get a stationary action metric under its requirements on the equator. The only possibility is a solution with some mild singularities within the extended class $C$.

It has been proven [10] that a stationary action regular solution keeps its status under the extension of class $C$. This fact keeps the existing theory intact. A stationary action solution with some mild singularities satisfies the usual Einstein field equation except for those locations in the manifold where some conditions are imposed. For the black-hole cases considered in this paper, the Euclidean solutions satisfy the usual Einstein field equation, except for the singularities at the black hole and the cosmological horizons [6, 7]. The stationary action metric with some mild singularities can be called the generalized gravitational instanton. It can be considered as a solution of the generalized Einstein equation.
At the transition surface $\Sigma$, it is assumed that along neither of the sides of $\Sigma$ does a singular matter distribution exist. It follows from the Einstein equation that the second fundamental form $K_{ij}$ at $\Sigma$ should vanish,

$K_{ij} = 0$.

This condition cannot apply to the mild singularities at the black hole nor at cosmological horizons in our cases, since the usual Einstein equation does not hold there.

As we mentioned above, in quantum cosmology one uses a Lorentzian metric to join a Euclidean metric, both being sectors of a complex manifold. However, there exist very few complex manifolds satisfying the Einstein equation with both a Euclidean and a Lorentzian sector [11]. One may appeal to some approximately Euclidean or Lorentzian sectors, but only at the price of losing some of the beauty of the theory. In the extended framework the requirement becomes quite loose. The situation of black-hole creation we are going to investigate is the best illustration.

This is the third of a series of papers on quantum creation of a single black hole [6, 7]. The first paper considered the neutral and nonrotating black hole, i.e. the Schwarzschild black hole. The second is devoted to the charged but nonrotating black hole, i.e. the Reissner-Nordström black hole. The current paper will deal with the rotating black-hole case. Section 2 will review the nonrotating black-hole case. Section 3 is devoted to the rotating but neutral black-hole case, i.e. the Kerr black hole. Section 4 investigates the rotating and charged case, i.e. the Newman black hole. By the no-hair theorem, all these kinds of black holes have exhausted the stationary vacuum or electrovac cases. Therefore, the problem of quantum creation of a single black hole in quantum cosmology is completely resolved. Section 5 is a discussion.

2. - The spherically symmetric black hole

Let us begin with a quantum spherically symmetric vacuum or electrovac model with a positive cosmological constant $\Lambda$. The cosmological constant may be effective due to the Planckian inflation in the Hawking massive scalar model [12]. At the semiclassical level the evolution of the universe is described by its classical solutions. The Schwarzschild-de Sitter space-time with mass parameter $m$ and zero charge $Q$ is the unique spherically symmetric vacuum solution to the Einstein equation with a cosmological constant $\Lambda$. The Reissner-Nordström-de Sitter space-time, with mass parameter $m$, nonzero charge $Q$ and a cosmological constant $\Lambda$, is the only spherically symmetric electrovac solution to the Einstein and Maxwell equations. Its Euclidean metric can be written as [13]

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right) dr^2 +$$

$$+ \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$
We can set
\[
V_s = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}.
\]

For convenience one can make a factorization
\[
V_s = -\frac{\Lambda}{3r^2} (r - r_0)(r - r_1)(r - r_2)(r - r_3),
\]
where \(r_0, r_1, r_2, r_3\) are in ascending order. \(r_2\) and \(r_3\) are the black hole and cosmological horizons, where conical singularities may occur, \(r_0\) is negative. If the black hole is neutral, then \(r_3\) can be set to zero, and there are essentially three roots left.

The gauge field is
\[
F = -\frac{iQ}{r^2} \, d\tau \wedge dr
\]
for an electrically charged solution, and
\[
F = Q \sin \theta \, d\theta \wedge d\phi
\]
for a magnetically charged solution. We shall not consider dyonic solutions.

The roots satisfy the following relations:
\[
\sum \! r_i = 0,
\]
\[
\sum \! r_i r_j = -\frac{3}{\Lambda},
\]
\[
\sum \! r_i r_j r_k = -\frac{6m}{\Lambda}
\]
and
\[
\prod \! r_i = -\frac{3Q^2}{\Lambda}.
\]

The black hole and cosmological surface gravities \(\kappa_2\) and \(\kappa_3\) are [13]
\[
\kappa_2 = \frac{1}{2} \left| V'_s (r_2) \right| = \frac{\Lambda}{6r_2^2} (r_2 - r_0)(r_2 - r_1)(r_3 - r_2),
\]
\[
\kappa_3 = \frac{1}{2} \left| V'_s (r_3) \right| = \frac{\Lambda}{6r_3^2} (r_3 - r_0)(r_3 - r_1)(r_3 - r_2).
\]

The requirement of vanishing second fundamental form at \(\Sigma\) minus the two conical singularities at the two horizons implies that the transition can only occur at two sections of constant values of imaginary time \(\tau\) glued at the two horizons. The 3-surface \(\Sigma\) has topology \(S^2 \times S^1\). To form a generalized gravitational instanton, one can have two cuts at \(\tau = \text{const}\) between \(r = r_2\) and \(r = r_3\). Then the \(f_2\)-fold cover turns the \((\tau - r)\) plane
into a cone with a deficit angle $2\pi (1 - f_2)$ at the black-hole horizon. In a similar way one can have an $f_3$-fold cover at the cosmological horizon. Both $f_2$ and $f_3$ can take any pair of real numbers with the relation

$$f_2 \beta_2 = f_3 \beta_3,$$

where $\beta_2 = 2\pi K_2^{-1}$ and $\beta_3 = 2\pi K_3^{-1}$.

If $f_2$ or $f_3$ is different from 1, then the cone at the black hole or cosmological horizon will have an extra contribution to the action of the instanton. After the transition to Lorentzian space-time, the conical singularities will only affect the real part of the phase of the wave function, i.e. the probability of the creation of the black hole.

Since the integral of $K$ with respect to the 3-area in the boundary term of the action (2) is the area increase rate along its normal, then the extra contribution due to the conical singularities can be considered as the degenerate form shown below:

$$\tilde{I}_{2, \text{deficit}} = - \frac{1}{8\pi} \cdot 4\pi r_2^2 \cdot 2\pi (1 - f_2),$$

$$\tilde{I}_{3, \text{deficit}} = - \frac{1}{8\pi} \cdot 4\pi r_3^2 \cdot 2\pi (1 - f_3).$$

The action due to the volume is

$$\tilde{I}_v = - \frac{f_2 \beta_2 A}{6} (r_3^3 - r_2^3) \pm \frac{f_2 \beta_2 Q^2}{2} (r_2^{-1} - r_3^{-1}),$$

where $+$ is for the magnetic case and $-$ is for the electric case. This term disappears for the neutral case.

In the neutral case, the boundary date on the 3-surface $\Sigma$ will be $h_{ij}$. In the magnetic case, the boundary date is $h_{ij}$ and $A_{ij}$. The vector potential in turn determines the magnetic charge, since it can be obtained by the magnetic flux, or the integral of the gauge field $F$ over the $S^2$ space sector. It is more convenient to choose a gauge potential

$$A = Q(1 - \cos \theta) \, d\phi$$

to evaluate the flux.

In the electric case, the boundary date is $h_{ij}$ and the momentum $\omega$ [14], which is canonically conjugate to the electric charge and defined by

$$\omega = \int A,$$

where the integral is around the $S^1$ direction. The most convenient choice of the gauge potential for the calculation is

$$A = - \frac{iQ}{r^2} \, r \, dr.$$

The wave function for the equator is the exponential of half the negative of the action. For the neutral and magnetic cases, one obtains the wave function $\Psi(h_{ij})$ and
\[ \Psi(Q, h_{ij}) \]. For the electric case, what one obtains this way is \[ \Psi(\omega, h_{ij}) \] instead of \[ \Psi(Q, h_{ij}) \]. One can get the wave function \[ \Psi(Q, h_{ij}) \] for a given electric charge through the Fourier transformation [10, 14]

\[ \Psi(Q, h_{ij}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega e^{i\omega Q} \Psi(\omega, h_{ij}). \] (22)

This Fourier transformation is equivalent to a multiplication of an extra factor

\[ \exp \left[ \frac{-f_2 \beta_2 Q^2 (r_2^{-1} - r_3^{-1})}{2} \right] \] (23)

to the wave function. This makes the probabilities for magnetic and electric cases equal, and thus recovers the duality between the magnetic and electric black holes [14].

Finally, using the relations (15) and (9)-(12), one obtains the probability for a spherically symmetric black-hole creation

\[ P_s = \exp \{\pi(r_2^2 + r_3^2)\}. \] (24)

This is the exponential of one quarter of the sum of the black hole and cosmological horizon areas, or the total entropy of the universe.

The most remarkable fact is that the result is independent of our choice of \( f_2 \) or \( f_3 \). This means that our generalized gravitational instanton has a stationary action, and can be used for the WKB approximation to the wave function.

There have been many studies recently on the nonsingular, charged or neutral, spherically symmetric instantons and the associated black-hole creations [1-5]. All these instantons lead to the creation of pairs of black holes. For these cases one can avoid the conical singularities by choosing \( f_2 = f_3 = 1 \), since the two surface gravities are identical. However, their results are the special cases of our general formula (24), recalling that the degenerate horizon should be counted twice.

The wave function for the spherically symmetric black hole can also be found [6, 7]. When \( m = 0 \) and \( Q = 0 \), it is reduced to the de Sitter case

\[ P_0 = \exp \left[ \frac{3\pi}{\Lambda} \right] \] (25)

and when \( Q = 0 \) and \( r_2 = r_3 \), it is reduced to the Nariai case

\[ P_{nc} = \exp \left[ \frac{2\pi}{\Lambda} \right] \] (26)

The formula (24) interposes the above values for the two extreme cases of neutral black holes.

The probability is a decreasing function with respect to parameter \( m \) and \( |Q| \). So the de Sitter universe is the most probable one for the Planckian era in quantum cosmology, as is expected.
3. The Kerr-de Sitter black hole

Now let us discuss the creation of a rotating black hole in the de Sitter space background. The Lorentzian metric of the black-hole space-time is [13]

\[
\begin{align*}
\text{d}s^2 &= \varrho^2(\Delta_r^{-1}\text{d}r^2 + \Delta_\theta^{-1}\text{d}\theta^2) + \\
&+ \varrho^{-2}\Xi^{-2}\Delta_\phi\sin^2\theta(a\text{dt} - (r^2 + a^2)\text{d}\varphi)^2 - \varrho^{-2}\Xi^{-2}\Delta_r(\text{d}t - a\sin^2\theta\text{d}\varphi)^2,
\end{align*}
\]

where

\[
\begin{align*}
\varrho^2 &= r^2 + a^2\cos^2\theta, \\
\Delta_r &= (r^2 + a^2)(1 - \Lambda r^2 3^{-1}) - 2mr + Q^2 + P^2, \\
\Delta_\theta &= 1 + \Lambda a^2 3^{-1}\cos^2\theta, \\
\Xi &= 1 + \Lambda a^2 3^{-1}
\end{align*}
\]

and \(m, a, Q\) and \(P\) are constants, \(m\) and \(ma\) representing mass and angular momentum. \(Q\) and \(P\) are electric and magnetic charges.

One can factorize \(\Delta_r\) as follows:

\[
\Delta_r = -\frac{\Lambda}{3}(r - r_0)(r - r_1)(r - r_2)(r - r_3),
\]

where the roots \(r_0, r_1, r_2\) and \(r_3\) are in ascending order, \(r_2\) and \(r_3\) are the black hole and cosmological horizons. The roots satisfy the following relations:

\[
\begin{align*}
\sum_{i} r_i &= 0, \\
\sum_{i > j} r_i r_j &= -\frac{3}{\Lambda} + a^2, \\
\sum_{i > j > k} r_i r_j r_k &= -\frac{6m}{\Lambda}, \\
\prod_{i} r_i &= -\frac{3(a^2 + Q^2 + P^2)}{\Lambda}.
\end{align*}
\]

In this section we shall concentrate on the neutral case with \(Q = P = 0\). The Newman case with nonzero electric or magnetic charge will be differed to the next section.

The probability of the Kerr black-hole creation, at the WKB level, is the exponential of the negative half of its corresponding generalized gravitational instanton. The only instanton which can be used to join the Lorentzian sector at the quantum transition is the complex space-time obtained from the Lorentzian metric by a substitution \(t \rightarrow -i\pi\) only. However, for convenience of calculation, we can let \(a\) to be imaginary, and then the complex metric becomes Euclidean. After we get the probability for the imaginary \(a\) value, then we can analytically continue back to real \(a\) to obtain the required probability.
In order to form a generalized gravitational instanton, one can do the similar cutting, folding and covering at both the black hole and cosmological horizons with $f_2$ and $f_3$ satisfying relation (15) as in the nonrotating case. We shall freely switch back and forth between the real and imaginary values of $a$ in the following calculation to facilitate our interpretation.

For the Kerr case, the topology of 3-surface $\Sigma$ is $S^2 \times S^1$. Their horizon areas are

\begin{align}
A_2 &= 4\pi (r_2^2 + a^2) \Xi^{-1}, \\
A_3 &= 4\pi (r_3^2 + a^2) \Xi^{-1}.
\end{align}

The black-hole and cosmological surface gravities are

\begin{align}
\kappa_2 &= \frac{\Lambda(r_2 - r_0)(r_2 - r_1)(r_3 - r_2)}{6\Xi(r_2^2 + a^2)}, \\
\kappa_3 &= \frac{\Lambda(r_3 - r_0)(r_3 - r_1)(r_3 - r_2)}{6\Xi(r_3^2 + a^2)}.
\end{align}

The actions due to the conical singularities are

\begin{align}
\tilde{I}_{2, \text{deficit}} &= -\frac{\pi (r_2^2 + a^2)(1 - f_2)}{\Xi}, \\
\tilde{I}_{3, \text{deficit}} &= -\frac{\pi (r_3^2 + a^2)(1 - f_3)}{\Xi}.
\end{align}

The action due to the volume is

\begin{align}
\tilde{I}_v &= -\frac{f_2 \beta_2 \Lambda}{6\Xi^2} (r_3^3 - r_2^3 + a^2(r_3 - r_2)),
\end{align}

where $\beta_2$ is defined as before.

If one naively takes the exponential of the negative of half the total action (after the analytic continuation by the replacement of $b$ by $a$), then the wave function for the creation moment of a black hole with parameter $m$ and $a$ will not be obtained. The physical reason is that what one can observe is only the angular differentiation, or the relative rotation of the two horizons. This situation is similar to the case of a Kerr black hole in the asymptotically flat background. There one can only measure the rotation of the black-hole horizon from the spatial infinity. To find the wave function for the given mass and angular momentum one has to make the Fourier transformation

\begin{align}
\Psi(m, a, h_{ij}) &= \frac{1}{2\pi} \int d\delta e^{\frac{i}{\Xi} \delta^2} \Psi(m, \delta, h_{ij}),
\end{align}

where $\delta$ is the relative rotation angle for the time period $f_2 \beta_2$, which is canonically conjugate to the angular momentum $J \equiv ma$; and the factor $\Xi^{-2}$ is due to the time
rescaling. The angle difference $\Delta$ can be evaluated:

$$\Delta = \int_0^{f_2/\beta_2} d\tau (\Omega_2 - \Omega_3),$$

where the angular velocities at the two horizons are

$$\Omega_2 = \frac{a}{r_2^2 + a^2},$$

and

$$\Omega_3 = \frac{a}{r_3^2 + a^2}.$$  

The Fourier transformation is equivalent to adding an extra term into the action for the generalized instanton, and then the total action becomes

$$\tilde{I} = -\pi (r_2^2 + a^2) \Xi^{-1} - \pi (r_3^2 + a^2) \Xi^{-1}$$

and the probability of the Kerr black-hole creation is

$$P_k \approx \exp \left[ \pi (r_2^2 + a^2) \Xi^{-1} + \pi (r_3^2 + a^2) \Xi^{-1} \right].$$

It is the exponential of one quarter of the two horizon areas, or the total entropy of the universe. One notices that our generalized gravitational instanton has a stationary action and can be used for the WKB approximation of the wave function. This will remain true for the Newman case below.

4. - The Newman-de Sitter black hole

Now let us turn to the charged black-hole case. The vector potential can be written as

$$A = \frac{Qr (dt - a \sin^2 \theta \, d\phi) + P \cos \theta (a \, dt - (r^2 + a^2) \, d\phi)}{\theta^2}. $$

We shall not consider the dyonic case below. One can closely follow the neutral rotating case for calculating the action of the corresponding generalized gravitational instanton. The only difference is to add one more term due to the electromagnetic field to the action of volume. For the magnetic case, it is

$$\frac{f_2 \beta_2 P^2}{2 \Xi^2} \left( \frac{r_2}{r_2^2 + a^2} - \frac{r_3}{r_3^2 + a^2} \right),$$

and for the electric case, it is

$$\frac{-f_2 \beta_2 Q^2}{2 \Xi^2} \left( \frac{r_2}{r_2^2 + a^2} - \frac{r_3}{r_3^2 + a^2} \right).$$
In the magnetic case the vector potential determines the magnetic charge, which is the integral over the $S^2$ space sector. Putting all these contributions together one can find

$$\hat{I} = -\pi(r_2^2 + a^2) \Xi^{-1} - \pi(r_3^2 + a^2) \Xi^{-1}$$

and the probability of the creation of a magnetically charged black hole is

$$P_n \approx \exp \left[ \pi(r_2^2 + a^2) \Xi^{-1} + \pi(r_3^2 + a^2) \Xi^{-1} \right].$$

In the electric case, one can only fix the integral

$$\omega = \int A,$$

where the integral is around the $S^1$ direction. So, what one obtains in this way is

$$\Psi(\omega, a, h_{ij}).$$

In order to get the wave function $\Psi(Q, a, h_{ij})$ for a given electric charge, we have to repeat the procedure like the Reissner-Nordstrøm case. The Fourier transformation is equivalent to adding one more term to the action

$$f_2 \beta Q^2 \Xi^2 \left( \frac{r_2}{r_2^2 + a^2} - \frac{r_3}{r_3^2 + a^2} \right).$$

Then we obtain the same formula for the electrically charged rotating black-hole creation as that for the magnetic one,

$$P_n \approx \exp \left[ \pi(r_2^2 + a^2) \Xi^{-1} + \pi(r_3^2 + a^2) \Xi^{-1} \right].$$

It is easy to show that the probability is an exponentially decreasing function of the mass parameter, charge magnitude and angular momentum, and the de Sitter space-time is the most probable Lorentzian evolution at the Planckian era.

5. - Discussion

The result of this paper is the confirmation of the conjecture made in ref. [5] for all kinds of single black holes in the de Sitter space background. The probability of the black-hole creation is the exponential of the total entropy of the universe. The entropy is equal to one quarter of the sum of the black-hole and cosmological horizon areas.

The probability is an exponentially decreasing function in terms of the mass parameter, charge magnitude and angular momentum. Since this is only the confirmation of the conjecture, the result is no surprise. The only surprise is the fact that our result is independent of the choice of $f_2$ or $f_3$ for the formation of the generalized gravitational instantons.

To get a meaningful result, one has to be careful to identify the meaning of the wave function; so for the rotating case and electrically charged black holes, one has to introduce Fourier transformations into the calculation; otherwise, the result becomes meaningless. It is interesting to note that Nature would give us a beautiful result if our request were reasonable.

In quantum field theory, the temperature associated with a black hole is well defined. By using the reciprocal of the Hawking temperature as the period of the imaginary time, one can avoid the conical singularity at the horizon. However, if we
remain only at thermodynamics level, and if one considers the reciprocal of the period for the generalized gravitational instanton as an effective temperature, then from the calculation it seems the temperature can be taken quite arbitrarily. We appear to overcome the obstacle that the temperature of the black hole and cosmological horizons, in general, are different. This makes our calculation feasible. Temperature is a very subtle concept even in special relativity, let alone in general relativity. A thorough discussion about temperature is beyond the scope of this paper. However, the concept of entropy is very clear in any case.

Our calculation has also very clearly shown that the gravitational entropy is associated with topology of space-time, as Hawking emphasized many times [15].

From the no-hair theorem, a stationary black hole in the de Sitter space-time background can only have three parameters, mass, charge and angular momentum, so the problem of the quantum creation of a single black hole at the birth of the universe is completely resolved.

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